

Magnetic moment of the pentaquark $\Theta^+(1540)$ as diquark–diquark–antiquark state with QCD sum rules

Z.G. Wang^{1,a}, S.L. Wan², W.M. Yang²

¹ Department of Physics, North China Electric Power University, Baoding 071003, P.R. China

² Department of Modern Physics, University of Science and Technology of China, Hefei 230026, P.R. China

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Abstract. In this article, we study the magnetic moment of the pentaquark state $\Theta^+(1540)$ as a diquark–diquark–antiquark ($[ud][ud\bar{s}]$) state with the QCD sum rules in an external weak electromagnetic field (EFSR) and the light-cone QCD sum rules (LCSR) respectively. The numerical results indicate that the magnetic moment is about $\mu_{\Theta^+} = -(0.11 \pm 0.02)\mu_N$ for the EFSR and $\mu_{\Theta^+} \approx -(0.1-0.5)\mu_N$ for the LCSR. As the values obtained from the EFSR are more stable than the corresponding ones from the LCSR, $\mu_{\Theta^+} = -(0.11 \pm 0.02)\mu_N$ is more reliable.

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1 Introduction

The observation of the new baryon state $\Theta^+(1540)$ with positive strangeness and minimal quark content, $ududs$ [1], has motivated intense theoretical investigations to clarify the quantum numbers and to understand the substructures of the exotic state [2, 3]. Although the pentaquark state $\Theta^+(1540)$ can be assigned to the top of the antidecuplet $\bar{10}$ with isospin $I = 0$, the spin and parity have not been experimentally determined yet and no consensus has ever been reached on the theoretical side [2, 3]. The discovery has opened a new field of the strong interactions and provides a new opportunity for a deeper understanding of low energy QCD, especially when multi-quark states are involved. The magnetic moments of the pentaquark states are fundamental parameters, as their masses, which have copious information about the underlying quark structures, can be used to distinguish the preferred configurations from various theoretical models and deepen our understanding of the underlying dynamics. Furthermore, the magnetic moment of the pentaquark state $\Theta^+(1540)$ is an important ingredient in studying the cross sections of the photo- or electro-production, which can be used to determine the fundamental quantum numbers of the pentaquark state $\Theta^+(1540)$, such as spin and parity [4, 8], and may be extracted from experiments eventually in the future.

There have been several works on the magnetic moments of the pentaquark state $\Theta^+(1540)$ [5–15]; in this article, we take the point of view that the baryon $\Theta^+(1540)$ is a diquark–diquark–antiquark ($[ud][ud\bar{s}]$) state with the quantum numbers $J = \frac{1}{2}$, $I = 0$, $S = +1$, and study its

magnetic moment with the QCD sum rules in the external weak electromagnetic field (EFSR) and the light-cone QCD sum rules (LCSR) respectively [16–18]. Different quark configurations can be implemented with different interpolating currents, and if the u and d quarks in the pentaquark state $\Theta^+(1540)$ are bound into spin zero, color and flavor antitriplet $\bar{3}$ diquarks, we can take the diquarks (for example, $\epsilon^{abc}u_b C\gamma_5 d_c$ with $J^P = 0^+$ and $\epsilon^{abc}u_b C d_c$ with $J^P = 0^-$) instead of the u and d quarks as the basic constituents to construct the interpolating currents.

The article is arranged as follows: we derive the EFSR and LCSR for the magnetic moment of the pentaquark state $\Theta^+(1540)$ in Sect. 2; in Sect. 3, numerical results and discussions; Sect. 4 is reserved for our conclusion.

2 EFSR and LCSR

Although for medium and asymptotic momentum transfers the operator product expansion approach can be applied for the form factors and moments of the wave functions, at low momentum transfer, the standard operator product expansion approach cannot be consistently applied, as pointed out in the early work on photon couplings at low momentum for the nucleon magnetic moments [17]. In [17], the problem was solved by using a two-point correlation function in an external electromagnetic field, with vacuum susceptibilities introduced as parameters for the non-perturbative propagation in the external field, i.e. the QCD sum rules in the external field. As non-perturbative vacuum properties, the susceptibilities can be introduced for both small and large momentum transfers in the external fields. The alternative way is the light-cone QCD

^a e-mail: wangzgyiti@yahoo.com.cn

sum rules, which was firstly used to calculate the magnetic moments of the nucleons in [19]. For more discussions of the magnetic moments of the baryons in the framework of the LCSR approach, one can consult [20].

In the following, we write down the two-point correlation functions $\Pi_{\text{EF}\eta}(p)$ and $\Pi_{\text{LC}\eta}(p)$ for the EFSR and LCSR respectively [21],

$$\begin{aligned}\Pi_{\text{EF}\eta}(p) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ \eta(x) \bar{\eta}(0) \} | 0 \rangle_{F_{\mu\nu}}, \\ &= \Pi_0(p) + \Pi_{\mu\nu}(p) F^{\mu\nu} + \dots,\end{aligned}\quad (1)$$

$$\Pi_{\text{LC}\eta}(p, q) = i \int d^4x e^{ipx} \langle \gamma(q) | T \{ \eta(x) \bar{\eta}(0) \} | 0 \rangle, \quad (2)$$

where

$$\begin{aligned}\eta_1(x) &= \frac{1}{\sqrt{2}} \epsilon^{abc} \{ [u_a^T(x) C \gamma_5 d_b(x)] [u_c^T(x) C \gamma_5 d_e(x)] C \bar{s}_e^T(x) \\ &\quad - (u \leftrightarrow d) \},\end{aligned}\quad (3)$$

$$\begin{aligned}\eta_2(x) &= \frac{1}{\sqrt{2}} \epsilon^{abc} \{ [u_a^T(x) C d_b(x)] [u_c^T(x) C d_e(x)] C \bar{s}_e^T(x) \\ &\quad - (u \leftrightarrow d) \},\end{aligned}\quad (4)$$

$$\eta(x) = \{ t \eta_1(x) + \eta_2(x) \}. \quad (5)$$

Here the $\gamma(q)$ represents the external electromagnetic field $A_\mu(x) = \varepsilon_\mu e^{iq \cdot x}$, the ε_μ is the photon polarization vector and the field strength $F_{\mu\nu}(x) = i(\varepsilon_\nu q_\mu - \varepsilon_\mu q_\nu) e^{iq \cdot x}$. The $\Pi_0(p)$ is the correlation function without the external field $F_{\mu\nu}$ and $\Pi_{\mu\nu}(p)$ is the linear response term. The a, b, c and e are color indexes, the $C = -C^T$ is the charge conjugation operator, and t is an arbitrary parameter. The constituents $\epsilon^{abc} u_b^T(x) C \gamma_5 d_c(x)$ represent the scalar diquarks with $J^P = 0^+$ and $\epsilon^{abc} u_b^T(x) C d_c(x)$ represent the pseudoscalar diquarks with $J^P = 0^-$, we can denote the $\eta_1(x)$ and $\eta_2(x)$ by S-type and P-type interpolating current respectively according to the spin and parity of the constituent diquarks. They both belong to the antitriplet $\bar{3}$ representation of the color and flavor $SU(3)$ group, and can cluster together with diquark–diquark–antiquark structure to give the total spin and parity for the pentaquark state $\Theta^+(1540)$ $J^P = \frac{1}{2}^{+1}$. The scalar diquarks correspond to the 1S_0 states of the ud quark system. The one-gluon exchange force

¹ We can write down the interpolating currents for the other pentaquark states in the multiplets $\bar{10} + 8$ based on the Jaffe–Wilczek’s diquark model in the same way as we have done in (3)–(5), then perform the operator product expansion and use current–hadron duality to obtain the magnetic moments. Comparing with the magnetic moments in the multiplets and detailed studies may shed light on the substructures and low energy dynamics of the pentaquark states. However, the calculations of the operator product expansions for a number of correlation functions are tedious and beyond the present work; this may be our next work.

and the instanton induced force can lead to significant attractions between the quarks in the 0^+ channels [22]. The pseudoscalar diquarks do not have a non-relativistic limit, and can be taken as the 3P_0 states.

At the level of hadronic degrees of freedom, the linear response term $\Pi_{\mu\nu}(p)$ can be written as

$$\begin{aligned}\Pi_{\mu\nu}(p) F^{\mu\nu} &= i \int d^4x e^{ip \cdot x} \left\langle 0 | \eta(x) \right. \\ &\quad \left. \times \left\{ -i \int d^4y A_\mu(y) J^\mu(y) \right\} \bar{\eta}(0) | 0 \right\rangle.\end{aligned}\quad (6)$$

According to the basic assumption of current–hadron duality in the QCD sum rules approach [16], we insert a complete series of intermediate states satisfying the unitarity principle with the same quantum numbers as the current operator $\eta(x)$ into the correlation functions in (2) and (6) to obtain the hadronic representation. After isolating the double-pole terms of the lowest pentaquark states, we get the following results:

$$\begin{aligned}\Pi_{\mu\nu}(p) F^{\mu\nu} &= - \int d^4x \int d^4y \frac{d^4k}{(2\pi)^4} \frac{d^4k'}{(2\pi)^4} \\ &\quad \times \sum_{ss'} \frac{1}{m_{\Theta^+}^2 - k^2 - i\epsilon} \frac{1}{m_{\Theta^+}^2 - k'^2 - i\epsilon} \\ &\quad \times e^{ip \cdot x} A_\mu(y) \langle 0 | \eta(x) | ks \rangle \langle ks | J^\mu(y) | k' s' \rangle \langle k' s' | \bar{\eta}(0) | 0 \rangle \\ &\quad + \dots \\ &= -F^{\mu\nu} f_0^2 \frac{F_1(0) + F_2(0)}{4(m_{\Theta^+}^2 - p^2)^2} \{ \hat{p} \sigma_{\mu\nu} + \sigma_{\mu\nu} \hat{p} \} + \dots;\end{aligned}\quad (7)$$

$$\begin{aligned}\Pi_{\text{LC}\eta}(p, q) &= -f_0^2 \varepsilon^\mu \frac{\hat{p} + m_{\Theta^+}}{p^2 - m_{\Theta^+}^2} \left[F_1(q^2) \gamma_\mu + \frac{i \sigma_{\mu\nu} q^\nu}{2m_{\Theta^+}} F_2(q^2) \right] \\ &\quad \times \frac{\hat{p} + \hat{q} + m_{\Theta^+}}{(p+q)^2 - m_{\Theta^+}^2} + \dots \\ &= -\frac{f_0^2 [F_1(q^2) + F_2(q^2)]}{(p^2 - m_{\Theta^+}^2)((p+q)^2 - m_{\Theta^+}^2)} \hat{p} \hat{\varepsilon}(\hat{p} + \hat{q}) + \dots \\ &= \Pi_{\text{LC}}(p, q) i \varepsilon_{\mu\nu\alpha\beta} \gamma_5 \gamma^\mu \varepsilon^\nu q^\alpha p^\beta + \dots\end{aligned}\quad (8)$$

Here we have used the fix-point gauge $x_\mu A^\mu(x) = 0$, $A_\mu(y) = -\frac{1}{2} F_{\mu\nu} y^\nu$, and the definition $\langle 0 | \eta(0) | \Theta^+(p) \rangle = f_0 u(p)$. From the electromagnetic form factors $F_1(q^2)$ and $F_2(q^2)$, we can obtain the magnetic moment of the pentaquark state $\Theta^+(1540)$,

$$\mu_{\Theta^+} = \{ F_1(0) + F_2(0) \} \frac{e_{\Theta^+}}{2m_{\Theta^+}}. \quad (9)$$

The linear response term $\Pi_{\mu\nu}(p)$ in the weak external electromagnetic field $F_{\mu\nu}$ has three different Dirac tensor structures,

$$\Pi_{\mu\nu}(p) = \Pi_{\text{EF}}(p) \{ \sigma_{\mu\nu} \hat{p} + \hat{p} \sigma_{\mu\nu} \} \quad (10)$$

$$+ \Pi_1(p) i \{p_\mu \gamma_\nu - p_\nu \gamma_\mu\} \hat{p} + \Pi_2(p) \sigma_{\mu\nu}.$$

The first structure has an odd number of γ -matrices and conserves chirality, the second and third ones have an even number of γ -matrices and violate chirality. In the original QCD sum rules analysis of the nucleon magnetic moments [17], the interval of dimensions (of the condensates) for the odd structure is larger than the interval of dimensions for the even structures, and one may expect a better accuracy of the results obtained from the sum rules with the odd structure. In this article, the spin of the pentaquark state $\Theta^+(1540)$ is supposed to be $\frac{1}{2}$, just like the nucleon. As in our previous work [15], we can choose the first Dirac tensor structure $\{\sigma_{\mu\nu} \hat{p} + \hat{p} \sigma_{\mu\nu}\}$ for our analysis. The phenomenological spectral density of the EFSR in (7) can written

$$\frac{\text{Im}\Pi_{\text{EF}}(s)}{\pi} = \frac{1}{4} \{F_1(0) + F_2(0)\} f_0^2 \delta'(s - m_{\Theta^+}^2) + C_{\text{subtract}} \delta(s - m_{\Theta^+}^2) + \dots, \quad (11)$$

where the first term corresponds to the magnetic moment of the pentaquark state $\Theta^+(1540)$, and is of a double-pole nature. The second term comes from the electromagnetic transitions between the pentaquark state $\Theta^+(1540)$ and the excited states (or high resonances), and is of a single-pole character. Here we introduce the quantity C_{subtract} to represent the electromagnetic transitions between the ground pentaquark state and the high resonances; it may have a complex dependence on the energy s and high resonance masses. However, we have no knowledge about the high resonances, and even the existence of the ground pentaquark state $\Theta^+(1540)$ is not firmly established, which is in contrast to the conventional baryons. In those channels we can use the experimental data as a guide in constructing the phenomenological spectral densities [23]. In practical manipulations, we can take the C_{subtract} as an unknown constant, and, fitted to reproduce reliable values for the form factors $F_1(0) + F_2(0)$, we will revisit this subject in (19). The contributions from the higher resonances and continuum states are suppressed after a Borel transform and not shown explicitly for simplicity. For the LCSR, we write down only the double-pole term explicitly in (8), which corresponds to the magnetic moment of the pentaquark state $\Theta^+(1540)$, and choose the tensor structure $\varepsilon_{\mu\nu\alpha\beta} \gamma_5 \gamma^\mu \varepsilon^\nu q^\alpha p^\beta$ for the analysis [5]. The contributions from the single-pole terms which concern the excited and continuum states are suppressed after the double Borel transform, and are not shown explicitly for simplicity.

The calculation of the operator product expansion in the deep Euclidean space-time region at the level of quark and gluon degrees of freedom is straightforward and tedious; here technical details are neglected for simplicity, and once the analytical results are obtained, we can express the correlation functions at the level of quark–gluon degrees of freedom into the following forms through the dispersion relation

$$\Pi_{\text{EF}}(P^2) = \frac{e_s}{\pi} \int_{m_s^2}^{s_0} ds \frac{\text{Im}[A(s)]}{s + P^2} + e_s B(P^2) + \dots, \quad (12)$$

$$\begin{aligned} & \Pi_{\text{LC}}(p, q) \\ &= e_s \int_0^1 du \left\{ \frac{1}{\pi} \int_{m_s^2}^{s_0} ds \frac{\text{Im}[C(s)]}{s - up^2 - (1-u)(p+q)^2} \right. \\ & \quad \left. + D(p, q) \right\} + \dots, \end{aligned} \quad (13)$$

where

$$\begin{aligned} & \frac{\text{Im}[A(s)]}{\pi} \\ &= -\frac{(5t^2 + 2t + 5)s^4}{2^{14}5!4!\pi^8} + \frac{(5t^2 + 2t + 5)s^3 m_s \chi \langle \bar{s}s \rangle}{2^9 5!4!\pi^6} \\ & \quad + \frac{(5 + 2t - 7t^2)s \langle \bar{q}q \rangle^2}{2^9 3^2 \pi^4} \\ & \quad - \frac{(5 + 2t - 7t^2)m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \chi}{2^7 3^2 \pi^2} \\ & \quad - \frac{(5t^2 + 2t + 5)s^2}{2^{17}4!\pi^6} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle, \\ & B(P^2) = \frac{(5t^2 + 2t + 5) \langle \bar{q}q \rangle^4}{2^5 3^3 P^4}, \\ & \frac{\text{Im}[C(s)]}{\pi} \\ &= \frac{(5t^2 + 2t + 5)s^4}{2^{12}5!4!\pi^8} - \frac{(5t^2 + 2t + 5)s^3 f\psi(u)}{2^{13}5!\pi^6} \\ & \quad + \frac{(7t^2 - 2t - 5)s \langle \bar{q}q \rangle^2}{2^7 3^2 \pi^4} \\ & \quad - \frac{(7t^2 - 2t - 5) \langle \bar{q}q \rangle^2 f\psi(u)}{2^6 3 \pi^2} \\ & \quad + \frac{(5t^2 + 2t + 5)s^2}{2^{15}4!\pi^6} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \\ & \quad - \frac{(5t^2 + 2t + 5)s f\psi(u)}{2^{14}3\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle, \\ & D(p, q) = -\frac{(5t^2 + 2t + 5) \langle \bar{q}q \rangle^4}{2^3 3^3 (-up^2 - (1-u)(p+q)^2)^2}. \end{aligned}$$

From (12) and (13), we can see that due to the special interpolating current $\eta(x)$ (see (3)–(5)), the u and d quarks which constitute the diquarks have no contributions to the magnetic moment though they have electromagnetic interactions with the external field, and the net contributions to the magnetic moment come from the s quark only, which is significantly different from the results obtained in [5, 15], where all the u , d and s quarks have contributions. In [5, 15], the diquark–triquark type interpolating current $J(x)$ is used,

$$\begin{aligned} J(x) &= \frac{1}{\sqrt{2}} \epsilon^{abc} \{u_a^\top(x) C \gamma_5 d_b(x)\} \\ & \quad \times \{u_e(x) \bar{s}_e(x) i \gamma_5 d_c(x) - d_e(x) \bar{s}_e(x) i \gamma_5 u_c(x)\}. \end{aligned} \quad (14)$$

Although the diquark–diquark–antiquark type and diquark–triquark type configurations implemented by the interpolating currents $\eta(x)$ and $J(x)$ respectively can give satisfactory masses for the pentaquark state $\Theta^+(1540)$, and the resulting magnetic moments are substantially different. Once the magnetic moment can be extracted from the electro- or photo-production experiments, we can select the preferred configuration. In this article, we have neglected the contributions from the direct instantons as the effects are supposed to be small. In [24], the authors calculate the leading direct instanton contributions to the operator product expansion of the nucleon correlation function with the Ioffe current

$$J_p(x) = \epsilon^{abc} [u_a^T(x) C \gamma_\alpha u_b(x)] \gamma_5 \gamma^\alpha d_c(x)$$

in an external electromagnetic field, and find that the instanton contributions affect only the chiral odd sum rules which had previously been considered unstable. The general form of the proton current can be written [25]

$$J_p(x, t) = \epsilon_{abc} \left\{ [u_a^T(x) C d_b(x)] \gamma_5 u_c(x) + t [u_a^T(x) C \gamma_5 d_b(x)] u_c(x) \right\}, \quad (15)$$

in the limit $t = -1$, we recover the Ioffe current. In this article, we take the value of t to be $t = -1$ in (5). The pentaquark currents in (3)–(5) have a Dirac structure analogous to the baryon current in (15), so the contributions from the direct instantons may not affect significantly our analysis of the chiral even Dirac structure in (10). Furthermore, our previous work on the pentaquark mass using the interpolating current

$$J(x) = \epsilon^{abc} \epsilon^{def} \epsilon^{cfg} \{ u_a^T(x) C d_b(x) \} \times \{ u_d^T(x) C \gamma_5 d_e(x) \} C \bar{s}_g^T(x)$$

indicates that the direct instantons have negligible contributions [26]. The straightforward calculations and tedious analysis of the direct instanton contributions to the magnetic moment of the pentaquark state $\Theta^+(1540)$ will be our next work.

Here we will take a short digression and present some discussion of the condensates and light-cone amplitudes in (12) and (13). The presence of the external electromagnetic field $F_{\mu\nu}$ induces three new vacuum condensates i.e. the vacuum susceptibilities in the QCD vacuum [17]

$$\begin{aligned} \langle \bar{q} \sigma_{\mu\nu} q \rangle_{F_{\mu\nu}} &= e_q \chi F_{\mu\nu} \langle \bar{q} q \rangle, \\ g_s \langle \bar{q} G_{\mu\nu} q \rangle_{F_{\mu\nu}} &= e_q \kappa F_{\mu\nu} \langle \bar{q} q \rangle, \\ g_s \epsilon^{\mu\nu\lambda\sigma} \langle \bar{q} \gamma_5 G_{\lambda\sigma} q \rangle_{F_{\mu\nu}} &= i e_q \xi F^{\mu\nu} \langle \bar{q} q \rangle, \end{aligned}$$

where e_q is the quark charge, and χ , κ and ξ are the quark vacuum susceptibilities. The values with different theoretical approaches are different from each other; for a short review, see [28]. Here we shall adopt the values $\chi = -4.4 \text{ GeV}^{-2}$, $\kappa = 0.4$ and $\xi = -0.8$ [17, 18, 27]. In the calculation, we have neglected the terms which concern the $g_s \langle \bar{q} G_{\mu\nu} q \rangle$ and $g_s \epsilon^{\mu\nu\lambda\sigma} \langle \bar{q} \gamma_5 G_{\lambda\sigma} q | 0 \rangle$ induced vacuum

susceptibilities as they are suppressed by large denominators. The photons can couple to the quark lines perturbatively and non-perturbatively, which results in two classes of diagrams. In the first class of diagrams, the photons couple to the quark lines perturbatively through the standard QED; the second class of diagrams involve the non-perturbative interactions of photons with the quark lines, which are parameterized by the photon light-cone distribution amplitudes instead of the vacuum susceptibilities. In this article, the following two-particle photon light-cone distribution amplitude has contributions to the magnetic moment [18, 29],

$$\begin{aligned} &\langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle \\ &= \frac{f}{4} e_q \epsilon_{\mu\nu\rho\sigma} \epsilon^\nu q^\rho x^\sigma \int_0^1 du e^{iuqx} \psi(u), \quad (16) \end{aligned}$$

where the $\psi(u)$ is the twist-2 photon light-cone distribution amplitudes.

We make a Borel transform with respect to the variable P^2 in (12) and a double Borel transform with respect to the variables p^2 and $(p+q)^2$ in (13),

$$\Pi(M^2) \equiv \lim_{n, P^2 \rightarrow \infty} \frac{1}{\Gamma(n)} (P^2)^n \left(-\frac{d}{dP^2} \right)^n \Pi(P^2), \quad (17)$$

$$\begin{aligned} &\mathcal{B}_{(p+q)^2}^{M_1^2} \mathcal{B}_{p^2}^{M_2^2} \frac{\Gamma(n)}{[m^2 - (1-u)(p+q)^2 - up^2]^n} \\ &= (M^2)^{2-n} e^{-\frac{m^2}{M^2}} \delta(u - u_0), \quad (18) \end{aligned}$$

with $M^2 = P^2/n$ in (17) and $M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}$, $u_0 \equiv \frac{M_1^2}{M_1^2 + M_2^2}$ in (18). Finally we obtain the sum rules,

$$\{F_1(0) + F_2(0)\} (1 + CM^2) f_0^2 e^{-\frac{m_{\Theta^+}^2}{M^2}} = -4e_s AA, \quad (19)$$

$$\{F_1(0) + F_2(0)\} f_0^2 e^{-\frac{m_{\Theta^+}^2}{M^2}} = -e_s BB, \quad (20)$$

where

$$\begin{aligned} AA &= -\frac{(5t^2 + 2t + 5)M^{12}E_4}{2^{14}5!\pi^8} \\ &+ \frac{(5t^2 + 2t + 5)m_s \chi \langle \bar{s}s \rangle M^{10}E_3}{2^{11}5!\pi^6} \\ &+ \frac{(5 + 2t - 7t^2) \langle \bar{q}q \rangle^2 M^6 E_1}{2^9 3^2 \pi^4} \\ &- \frac{(5 + 2t - 7t^2)m_s \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \chi M^4 E_0}{2^7 3^2 \pi^2} \\ &- \frac{(5t^2 + 2t + 5)M^8 E_2}{2^{16}4!\pi^6} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \\ &+ \frac{(5t^2 + 2t + 5) \langle \bar{q}q \rangle^4}{2^5 3^3}, \\ BB &= \frac{(5t^2 + 2t + 5)M^{12}E_4}{2^{12}5!\pi^8} \end{aligned}$$

$$\begin{aligned}
& -\frac{(5t^2 + 2t + 5)M^{10}E_3f\psi(u_0)}{2^{15}5\pi^6} + \frac{(7t^2 - 2t - 5)\langle\bar{q}q\rangle^2 M^6 E_1}{2^7 3^2 \pi^4} \\
& -\frac{(7t^2 - 2t - 5)\langle\bar{q}q\rangle^2 f\psi(u_0)M^4 E_0}{2^6 3\pi^2} + \frac{(5t^2 + 2t + 5)M^8 E_2}{2^{14}4!\pi^6} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \\
& -\frac{(5t^2 + 2t + 5)f\psi(u_0)M^6 E_1}{2^{14}3\pi^4} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \\
& -\frac{(5t^2 + 2t + 5)\langle\bar{q}q\rangle^4}{2^3 3^3}, \\
E_n &= 1 - \exp\left(-\frac{s_0}{M^2}\right) \sum_{k=0}^n \left(\frac{s_0}{M^2}\right)^k \frac{1}{k!}.
\end{aligned}$$

The Borel transform in (17) cannot eliminate the contaminations to the correlation function from the single-pole terms, and we introduce the parameter C which is proportional to the C_{subtract} in (11) to subtract the contaminations. We have no knowledge of the electromagnetic transitions between the pentaquark state $\Theta^+(1540)$ and the excited states (or high resonances), and the C can be taken as a free parameter. We choose the suitable values for C to eliminate the contaminations from the single-pole terms to obtain reliable sum rules. The contributions from the single-pole terms may be as large as or larger than the double-pole term, and in practical calculations, the C can be fitted to give stable sum rules with respect to variations of the Borel parameter M^2 in a suitable interval. Taking the C as an unknown constant has smeared the complex energy s and high resonances masses dependence, which will certainly impair the predictive power. As final numerical results are insensitive to the threshold parameter s_0 and there really exists a platform with the variations of the Borel parameter M^2 , the predictions still make sense. The double Borel transform in (18) can eliminate the single-pole terms naturally; for more discussions of the double Borel transform, one can consult [30]. Furthermore, from the correlation function $\Pi_0(p)$ in (1), we can obtain the sum rules for the coupling constant f_0 [21],

$$\begin{aligned}
f_0^2 e^{-\frac{m_{\Theta^+}^2}{M^2}} &= CC, \tag{21} \\
CC &= \frac{3(5t^2 + 2t + 5)M^{12}E_5}{2^{11}7!\pi^8} \\
&+ \frac{(5t^2 + 2t + 5)m_s \langle\bar{s}s\rangle M^8 E_3}{2^{10}5!\pi^6} \\
&+ \frac{(1-t)^2 M^8 E_3}{2^{13}5!\pi^6} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \\
&+ \frac{(7t^2 - 2t - 5)\langle\bar{q}q\rangle^2 M^6 E_2}{2^9 3^2 \pi^4} \\
&- \frac{(5t^2 + 2t + 5)m_s \langle\bar{s}g_s\sigma Gs\rangle M^6 E_2}{2^{14}3^2 \pi^6}
\end{aligned}$$

$$\begin{aligned}
& + \frac{(7t^2 - 2t - 5)m_s \langle\bar{s}s\rangle \langle\bar{q}q\rangle^2 M^2 E_0}{2^6 3^2 \pi^2} \\
& + \frac{(5t^2 + 2t + 5)\langle\bar{q}q\rangle^4}{6^3}.
\end{aligned}$$

From the above equations, we can obtain the sum rules for the form factor $F_1(0) + F_2(0)$,

$$\{F_1(0) + F_2(0)\} \{1 + CM^2\} = -4e_s \frac{AA}{CC}, \tag{22}$$

$$\{F_1(0) + F_2(0)\} = -e_s \frac{BB}{CC}. \tag{23}$$

3 Numerical results and discussions

In this article, we take the value of the parameter t for the interpolating current $\eta(x)$ to be $t = -1$, which can give a stable mass for the pentaquark state $\Theta^+(1540)$ (i.e. $m_{\Theta^+} \approx 1540$ MeV) with respect to the variations of the Borel mass M^2 in the considered interval $M^2 = (2-3)$ GeV² [21]. The parameters for the condensates are chosen to be the standard values, although there are some suggestions for updating those values; for reviews, see [31]. We have $\langle\bar{s}s\rangle = (0.8 \pm 0.1)\langle\bar{u}u\rangle$, $\langle\bar{q}q\rangle = \langle\bar{u}u\rangle = \langle\bar{d}d\rangle = -(240 \pm 10)$ MeV³, $\langle\bar{s}g_s\sigma Gs\rangle = m_0^2 \langle\bar{s}s\rangle$, $m_0^2 = (0.8 \pm 0.1)$ GeV², $\chi = -(4.4 \pm 0.4)$ GeV⁻², $\langle\frac{\alpha_s GG}{\pi}\rangle = (0.33 \text{ GeV})^4$, $m_u = m_d = 0$ and $m_s = (140 \pm 10)$ MeV. Small variations of those condensates will not lead to large changes of the numerical values. The threshold parameter s_0 is chosen to vary between $(3.6-4.4)$ GeV² to avoid possible contaminations from the high resonances and continuum states, which is shown in Fig. 1 for $\langle\bar{s}s\rangle = 0.8\langle\bar{u}u\rangle$, $\langle\bar{q}q\rangle = -(240 \text{ MeV})^3$, $m_0^2 = 0.8 \text{ GeV}^2$, $\chi = -4.4 \text{ GeV}^{-2}$ and $m_s = 140 \text{ MeV}$. In the EFSR, for $s_0 = 4.0 \text{ GeV}^2$ and $M^2 = (2-5) \text{ GeV}^2$, we obtain the values

$$\begin{aligned}
F_1(0) + F_2(0) &= -(0.18 \pm 0.03), \\
\mu_{\Theta^+} &= -(0.18 \pm 0.03) \frac{e_{\Theta^+}}{2m_{\Theta^+}}, \\
&= -(0.11 \pm 0.02)\mu_N, \tag{24}
\end{aligned}$$

where μ_N is the nucleon magneton. From Table 1, we can

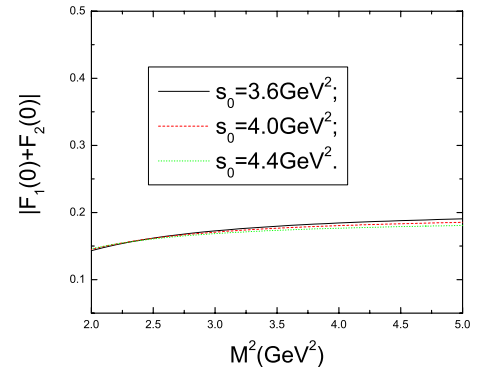


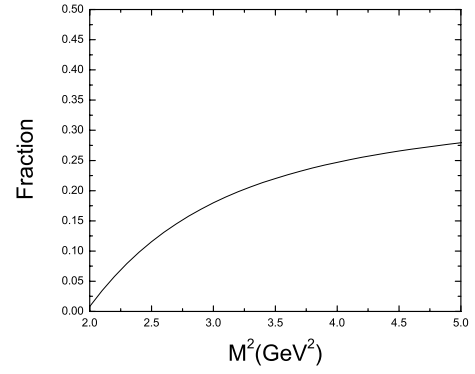
Fig. 1. $|F_1(0) + F_2(0)|$ with the variations of the Borel Parameter M^2 for EFSR

Table 1. The values of μ_{Θ^+} (in units of μ_N)

Reference	μ_{Θ^+} (μ_N)
[5]	0.12 ± 0.06
[6]	$0.08 \sim 0.6$
[7]	$0.2 \sim 0.3$
[8]	$0.2 \sim 0.5$
[9]	0.08 or 0.23 or 0.19 or 0.37
[10]	0.4
[11]	0.38
[12]	-1.19 or -0.33
[13]	0.71 or 0.56
[14]	0.362
[15]	0.24 ± 0.02
This work	$-(0.11 \pm 0.02)$

see that although the numerical values for the magnetic moment of the pentaquark state $\Theta^+(1540)$ vary with the theoretical approaches, they are small in general; our numerical results are consistent with most of the existing values of the theoretical estimates in magnitude, however, with negative sign. In (22), the perturbative contributions are about 25%, the dominating contributions come from the dimension-6 quark condensates terms $\langle \bar{q}q \rangle^2$, the contributions from the gluon condensate $\langle \frac{\alpha_s}{\pi} GG \rangle$, and the dimension-12 quark condensates, $\langle \bar{q}q \rangle^4$, are very small and can be safely neglected. In the calculation, other vacuum condensates are neglected due to the suppression of the large denominators, and the truncation of the operator product expansion makes sense.

For the conventional ground state mesons and baryons, due to the resonance dominating over the QCD continuum contributions, the good convergence of the operator product expansion, and the useful experimental guidance on the threshold parameter s_0 , we can obtain the fiducial Borel mass region. However, in the QCD sum rules for the pentaquark states, the spectral density $\rho(s) \sim s^m$ with m larger than the corresponding ones in the sum rules for the conventional baryons, a larger m means a stronger dependence on the continuum or the threshold parameter s_0 [32, 33]. In (21), due to the huge continuum contributions, the predicted mass increases with the continuum threshold s_0 , the QCD sum rules cannot strictly indicate the existence of the resonance in the spectral function, and the threshold parameter s_0 has to be fixed ad hoc or intuitively. In the finite energy sum rules (FESR) approach, the exponential weight function $\exp(-\frac{s}{M^2})$ is replaced by s^n in the numerical analysis. The FESR correlate the ground state mass and the QCD continuum threshold s_0 , and separate the ground state and QCD continuum contributions from the very beginning, and for some pentaquark currents, there happen to exist reasonable stability regions s_0 [32]. The weight function s^n enhances the continuum or the high mass resonances rather than the lowest ground state; we must make sure that only the lowest pole terms contribute

**Fig. 2.** $|\frac{[\frac{AA}{CC}]_{s_0=20.0 \text{ GeV}^2} - [\frac{AA}{CC}]_{s_0=4.0 \text{ GeV}^2}}{[\frac{AA}{CC}]_{s_0=4.0 \text{ GeV}^2}}|$ with the variations of the Borel parameter M^2 for EFSR

to the FESR below the s_0 – in some case, a naive stability region s_0 cannot guarantee a physically reasonable value for the s_0 [33]. It is obvious that, as QCD models, the Laplace sum rules and the FESR have both advantages and shortcomings. Although the quantities $f_0^2 \exp(-\frac{m_{\Theta^+}^2}{M^2})$ in (19) and (21) have a strong dependence on the continuum threshold parameter s_0 , the dependence is eliminated in (22) which results in a net s_0 insensitivity; it is far from the ideal case, where the $f_0^2 \exp(-\frac{m_{\Theta^+}^2}{M^2})$ are insensitive to the threshold parameter s_0 . From Fig. 2, we can see that for the Borel parameter $M^2 = (2-5) \text{ GeV}^2$, the contributions of the double-pole and single-pole terms below the threshold $s_0 = 4.0 \text{ GeV}^2$ are dominating; for example, the contributions of the continuum from the $s_0 = 4.0 \text{ GeV}^2$ to a large interval with $s_0 = 20.0 \text{ GeV}^2$ will not exceed 30% (comparing with the contributions below $s_0 = 4.0 \text{ GeV}^2$), the lowest pole terms dominance still hold and we have the fiducial Borel mass region where the neglect of higher-order terms in the short-distance expansion is justified while the nucleon pole terms still dominate over the continuum.

In the EFSR, the Borel transform cannot eliminate the contributions from the single-pole terms; on the other hand, we have no knowledge about the transitions between the ground states, the $\Theta^+(1540)$ state and excited states (or high resonances); in practical manipulations, we can introduce some free parameters which denote the contributions from the single-pole terms and subtract them. In choosing the parameter C in (22), we must take care; in general, the C can be chosen to give stable sum rules with respect to the variations of the Borel parameter M^2 . Although the uncertainty of the condensates, the neglect of the higher dimension condensates, the lack of perturbative QCD corrections, etc., will result in errors, we have stable sum rules for the magnetic moment, and furthermore, small variations of the condensates will not result in large changes for the values: the predictions are qualitative at least. If the pentaquark state $\Theta^+(1540)$ can be interpolated by the linear superposition of the S-type and P-type diquark–diquark–antiquark current, $\eta(x)$, the predictions are quantitative and make sense. Comparing with [15], the EFSR with different interpolating currents (standing for different configurations) can lead to very different results

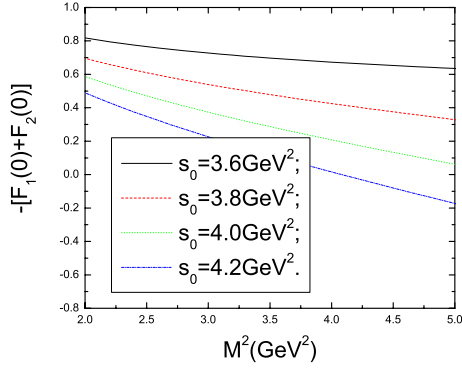


Fig. 3. $-[F_1(0) + F_2(0)]$ with the variations of the Borel parameter M^2 for LCSR

for the magnetic moment, although they can both give satisfactory masses for the pentaquark state $\Theta^+(1540)$.

In the LCSR, the double Borel transform is supposed to eliminate the single-pole terms which concern the contaminations from high resonances and continuum states, and gives stable sum rules with the variations of the Borel parameter M^2 . However, that is not the case; from Fig. 3, we can see that the LCSRs for the form factor $-[F_1(0) + F_2(0)]$ are sensitive to the threshold parameter s_0 , when $s_0 > 4.0 \text{ GeV}^2$, the values of the form factor $-[F_1(0) + F_2(0)]$ decrease drastically with the increase of the Borel parameter M^2 and there will not appear a platform. For $s_0 = 3.8 \text{ GeV}^2$, $M^2 = (2-5) \text{ GeV}^2$, $\psi(u) = 1$ and $f = 0.028 \text{ GeV}^2$ [18, 29], we have

$$\begin{aligned} F_1(0) + F_2(0) &= -(0.2-0.8), \\ \mu_{\Theta^+} &= -(0.2-0.8) \frac{e_{\Theta^+}}{2m_{\Theta^+}}, \\ &\approx -(0.1-0.5)\mu_N. \end{aligned} \quad (25)$$

In the above equations, we take the maximal variation interval for the values of the form factor $F_1(0) + F_2(0)$.

In the LCSR approach, the uncertainty of the photon light-cone distribution amplitudes, keeping only the lowest-twist few terms of the two particles distribution amplitudes, the uncertainty of the condensates, the neglect of the higher dimension condensates, the lack of perturbative QCD corrections, etc., can lead to errors in the predictions. The LCSRs for the form factor $F_1(0) + F_2(0)$ (see (23) and (25)) are very sensitive to the values of the parameter f , and small variations of the f can lead to large changes for the magnetic moment, which is shown in Fig. 4. The inclusion of the contributions from the direct instantons may improve the stability of the LCSR; to our knowledge, there exist no such type of works on the magnetic moments of the baryon with the LCSR; this may be our next work. In this article, only the non-perturbative interactions of the photons with the s quark line of the form $\langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle$ have contributions to the magnetic moment, which is significantly different from the corresponding ones with the interpolating current $J(x)$ in (14), where all the non-local matrix elements $\langle \gamma(q) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle$, $\langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle$, and $\langle \gamma(q) | \bar{q}(x) \gamma_\mu q(0) | 0 \rangle$ have contributions to the magnetic moment, and result in more stable sum rules [5]. In the vector

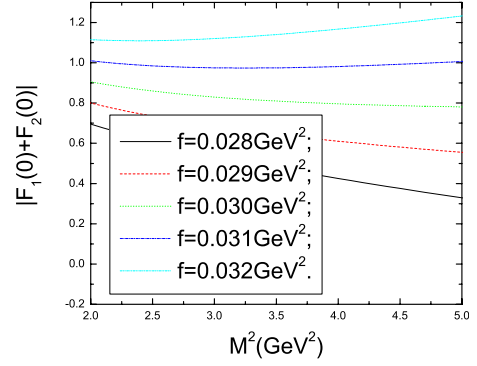


Fig. 4. $|F_1(0) + F_2(0)|$ for different f with the variations of the Borel parameter M^2 for LCSR ($s_0 = 3.8 \text{ GeV}^2$)

dominance model,

$$f \simeq \frac{f_\rho m_\rho}{g_\rho} \simeq 0.028 \text{ GeV}^2, \quad (26)$$

with $g_\rho = 5.5$ and $f_\rho = 0.2 \text{ GeV}$. To the leading-twist accuracy, the light-cone amplitude $\psi(u)$ is a constant which is set to unity due to the normalization condition.

We choose different tensor structures for the analysis in different approaches, i.e. the EFSR and the LCSR with the same interpolating current $\eta(x)$; the resulting different sum rules always lead to different predictions [5, 15]. The LCSRs with the interpolating current $\eta(x)$ depend heavily on the values of the parameter f and are sensitive to the threshold parameter s_0 – the values obtained from the LCSR (see (25)) are not as reliable as the corresponding ones from the EFSR (see (24)). When the experimental measurement of the magnetic moment of the pentaquark state $\Theta^+(1540)$ is possible in the future, we might be able to test the theoretical predictions, and select the preferred quark configurations.

4 Conclusion

In summary, we have calculated the magnetic moment of the pentaquark state $\Theta^+(1540)$ as a diquark–diquark–antiquark ($[ud][ud]\bar{s}$) state with both the EFSR and LCSR. We choose different tensor structures for our analysis in different approaches (i.e. the EFSR and the LCSR) with the same interpolating current, and the resulting different sum rules always lead to different predictions [5, 15]. The EFSRs for the magnetic moment are stable with the variations of the Borel parameter M^2 and insensitive to the threshold parameter s_0 (see Figs. 1 and 2); the LCSRs in this work heavily depend on the values of the non-local matrix element $\langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle$; small variations of the values of f can result in large changes of the magnetic moments. The values from the EFSR are more reliable than the corresponding ones from the LCSR. The numerical results from the EFSR are consistent with most of the existing values of the theoretical estimations in magnitude, however, with negative sign, $\mu_{\Theta^+} = -(0.18 \pm 0.03) \frac{e_{\Theta^+}}{2m_{\Theta^+}} = -(0.11 \pm 0.02)\mu_N$. Comparing with [15], the EFSR with dif-

ferent interpolating currents (standing for different configurations) can lead to very different results for the magnetic moment, although they can both give satisfactory masses for the pentaquark state $\Theta^+(1540)$. If the pentaquark state $\Theta^+(1540)$ can be interpolated by a linear superposition of both S-type and P-type diquark–diquark–antiquark current $\eta(x)$, the predictions are quantitative and make sense. The magnetic moments of the baryons are fundamental parameters as their masses, which have copious information about the underlying quark structures, and different substructures can lead to very different results. The width of the pentaquark state $\Theta^+(1540)$ is so narrow, that when the small magnetic moment can be extracted from electro- or photo-production experiments eventually in the future, this may be used to distinguish the preferred configurations from various theoretical models, and we may obtain more insight into the relevant degrees of freedom and deepen our understanding of the underlying dynamics that determines the properties of the exotic pentaquark states.

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